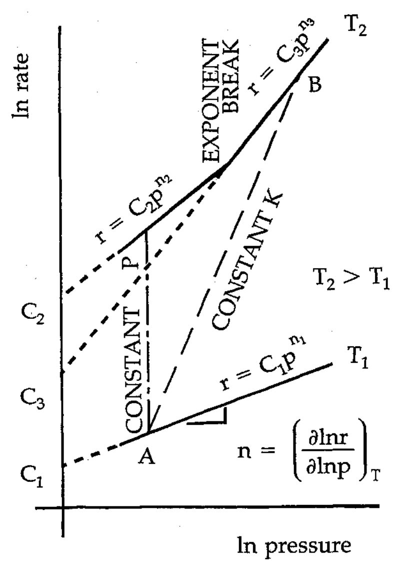

Fig. 2 Burning rate curves having exponent changes with pressure.

Thus, for Eq. (5), c and n not functions of pressure

$$\pi_K = \frac{1}{1-n}[\sigma_p + \pi_C]$$

$$= \frac{1}{1-n}\left[\left(\frac{\partial \ln c}{\partial T}\right)_p + \ln p\left(\frac{\partial n}{\partial T}\right)_p + \pi_C\right]$$

$$= \frac{1}{1-n_1}\left[\frac{\ln c_2 - \ln c_1}{T_2 - T_1} + \ln p_A\left(\frac{n_2 - n_1}{T_2 - T_1}\right) + \pi_C\right] \tag{15}$$

Similarly, for Eq. (10), c and n functions of both temperature *and* pressure

$$\pi_K = \frac{1}{1-n}\left[\left(\frac{\partial \ln c}{\partial T}\right)_K + \ln p\left(\frac{\partial n}{\partial T}\right)_K + \pi_C\right]$$

$$= \frac{1}{1-n_1}\left[\frac{\ln c_3 - \ln c_1}{T_2 - T_1} + \ln p_A\left(\frac{n_3 - n_1}{T_2 - T_1}\right) + \pi_C\right] \tag{16}$$

The difference between the two equations is clear; one [Eq. (15)] has partial derivatives with the combustion chamber pressure held constant, while the other [Eq. (16)] has partial derivatives with the motor geometry held constant. Therefore, for a propellant whose exponent changes with pressure, Eq. (16) represents the true value of the motor temperature sensitivity coefficient. Equation 15 does not.

Discussion

Two expressions for the motor temperature sensitivity coefficient, π_K, previously derived[3,4] have been shown to yield different results for the most general case in which the propellant is not well behaved, having either exponent breaks or gradual changes in exponent with pressure occur in its burning rate curves. The reason for the difference is that the one motor temperature sensitivity coefficient[3] [Eq. (5)] is dependent on the assumption that the burning rate parameters, c and n, are not functions of pressure. As a result, that motor temperature sensitivity coefficient[3] [Eq. (5)] cannot correctly represent combustion chamber pressure changes when exponent changes occur, when c and n are functions of temperature *and* pressure. More explicitly, the difference is that Eq. (5) cannot account completely for the total change in the combustion chamber pressure that occurs for a change in temperature of the solid rocket motor since Eq. (1) is a function of the propellant characteristics only and not the motor operating characteristics.

Conversely, the second motor temperature sensitivity coefficient[4] [Eq. (10)] has been derived for the case in which both burning rate parameters, c and n, are assumed to be functions of temperature *and* pressure. That coefficient[4] [Eq. (10)] involves the partial derivatives with the motor geometry held constant. It is, therefore, a true representation of the motor operation at the different temperatures for even the most general case in which the exponent changes with pressure.

Conclusions

The two expressions for the motor temperature sensitivity coefficient π_K were shown to be of equal complexity and ease of use. Both involve only the data taken for burning rate and use the parameters in St. Roberts' burning rate law. The most general expression for the motor temperature sensitivity coefficient [Eq. (10)] should be used since it is a true π_K relationship having the partial derivatives taken with the motor geometry held constant.

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Modeling of the Initial Droplet Size Distribution Function in the Spray Formed by Impinging Jets

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Introduction

THE modeling of thin sheets formed by impinging jets[1,2] and the ideas employed by Dombrowski[3] for the disin-

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tegration of viscous liquid sheets can be merged to allow the sizing and distribution of the droplets formed by colliding jets.[4]

This paper presents a method for the calculation of the initial droplet size distribution function and its moments (i.e., the Sauter mean diameter, the length mean diameter, the surface mean diameter, the surface-diameter mean, the volume mean diameter, the volume-diameter mean, and the geometric mean diameter), for, as it is well known, each one of them has its own preferred field of application. In particular the Sauter mean diameter, a parameter of wide application in the field of cumbustion, is shown to possess a minimum at a certain value of the impinging angle for several liquids.

Model Description

Consider two identical cylindrical liquid jets of radius R and velocity V_0 in a gaseous quiescent medium colliding obliquely at an angle 2θ, as sketched in Fig. 1. The sheet thus formed apreads away under the influence of the viscous, inertial, surface tension and pressure forces and, upon reaching a critical situation, disintegrates into fragments that contract themselves by surface tension, forming unstable ligaments that finally break into droplets.[3,4]

The number of droplets of a given spherical volume V_d generated per second, $d\dot{n}$, between ϕ and $\phi + d\phi$ (i.e., in the angular differential element taken relative to the separation streamline) can be written

$$d\dot{n} = \frac{d\dot{V}_E}{V_d} \tag{1}$$

where $d\dot{V}_E$ is the volume flow rate streaming out of the sector $d\phi$ as shown in Fig. 1. Assuming the same flow pattern as in Ref. 2 which implies that the mass flow rate is conserved within the above mentioned angular differential element, then

$$d\dot{V}_E = 2d\dot{V}_e \tag{2}$$

where the subscripts E and e stand for the sheet and the jet cross sections, respectively. Hence $d\dot{V}_e$ can be written

$$d\dot{V}_e = V_0 \sin\theta \frac{dp}{2} q \tag{3}$$

where q is the polar radius of the ellipse formed by the intersection of the jet with a plane parallel to the sheet referred to the separation point as a pole, and dp is the perimeter

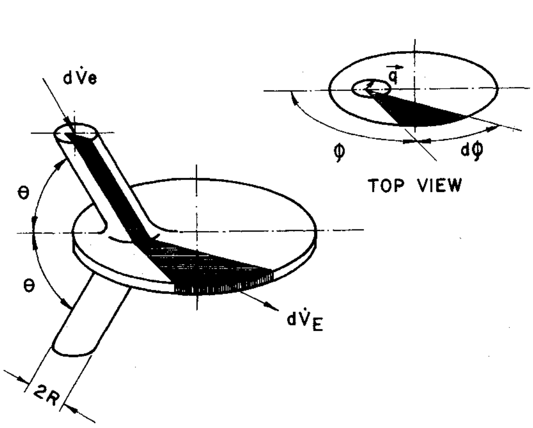

Fig. 1 Sheet formed by impinging jets.

Table 1 Mean diameters[5]

Mean diameter	Suggested field of application	Definition
Linear, $\bar{X}_a$	Evaporation	$\bar{X}_a = \frac{\Sigma x_{\phi i} f(\phi_i)\Delta\phi_i}{\Sigma f(\phi_i)\Delta\phi_{fi}}$
Surface, $\bar{X}_s$	Absorption, processes where surface area is controlling parameter	$\bar{X}_s = \sqrt{\frac{\Sigma x_{\phi i}^2 f(\phi_i)\Delta\phi_i}{\Sigma f(\phi_i)\Delta\phi_i}}$
Volume, $\bar{X}_v$	Comparison of mass distribution in a spray	$\bar{X}_v = \sqrt[3]{\frac{\Sigma x_{\phi i}^3 f(\phi_i)\Delta\phi_i}{\Sigma f(\phi_i)\Delta\phi_{fi}}}$
Surface diameter, $\bar{X}_{sd}$	Adsorption	$\bar{X}_{sd} = \frac{\Sigma x_{\phi i}^2 f(\phi_i)\Delta\phi_i}{\Sigma x_{\phi i} f(\phi_i)\Delta\phi_i}$
Volume diameter, $\bar{X}_{vd}$	Evaporation, molecular diffusion	$\bar{X}_{vd} = \sqrt{\frac{\Sigma x_{\phi i}^3 f(\phi_i)\Delta\phi_i}{\Sigma x_{\phi i} f(\phi_i)\Delta\phi_i}}$
Volume surface or Sauter diameter, $\bar{X}_{vs}$	Mass transfer, chemical reactions	$\bar{X}_{vs} = \frac{\Sigma x_{\phi i}^3 f(\phi_i)\Delta\phi_i}{\Sigma x_{\phi i}^2 f(\phi_i)\Delta\phi_i}$

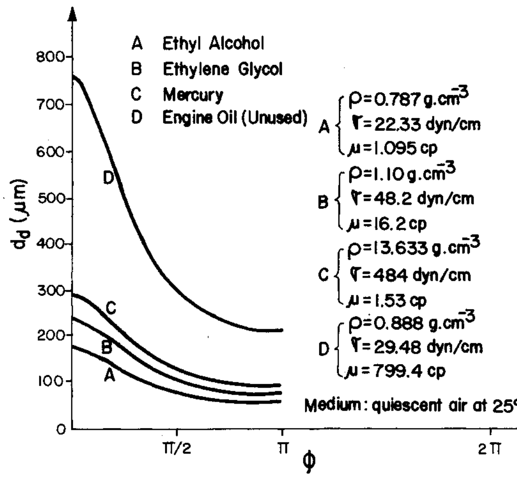

Fig. 2 Azimuthal droplet diameter distribution for a given configuration: θ = 45 deg, U = 3000 cm·s^{-1}, R = 0.05 cm (taken from Ref. 4).

differential element, written as

$$dp = \frac{R}{\sin\theta}\sqrt{1 - \cos^2\theta \sin^2\theta}\, d\phi \tag{4}$$

Further

$$q = \frac{R\sin\theta}{1 - \cos\theta\sin\phi} \tag{5}$$

Then substituting Eqs. (3), (4), and (5) in Eq. (2), one obtains

$$d\dot{V}_e = \frac{V_0 R^2 \sin\theta}{1 - \cos\theta\cos\phi}\sqrt{1 - \cos^2\theta\sin^2\phi}\, d\phi \tag{6}$$

and Eq. (1) becomes

$$d\dot{n} = \frac{6V_0R^2\sin\theta}{\pi d_d^3(1 - \cos\theta\cos\phi)}\sqrt{1 - \cos^2\theta\sin^2\phi}\, d\phi \tag{7}$$

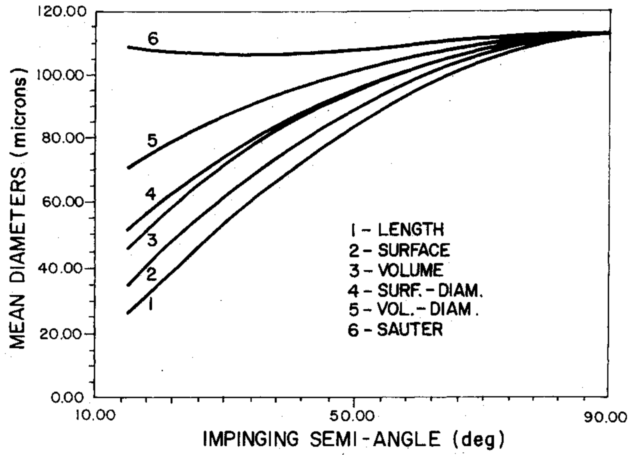

Fig. 3 Mean diameters [μm], versus impinging semiangle, θ [deg], for ethyl alcohol.

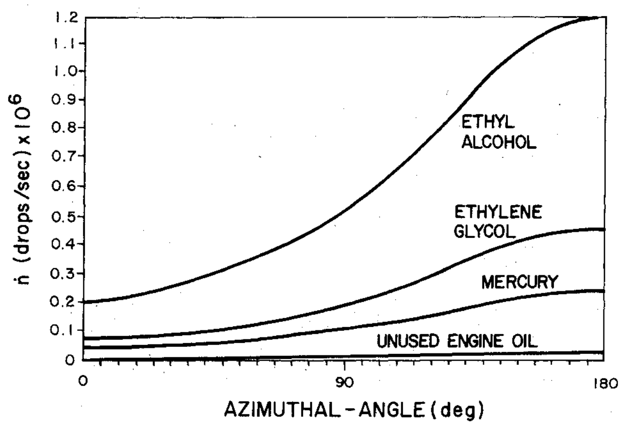

Fig. 4 Droplet formation rate, $\dot{n}$, vs azimuthal angle, ϕ [deg], for a given configuration (θ = 45 deg, U = 3000 cm $\cdot$ s^{-1}, R = 0.05 cm), in quiescent air at 25°C.

where d_d, the droplet diameter, can be written as[3]

$$d_d = \left(\frac{3\pi}{\sqrt{2}}\right)^{1/3} d_L \left[1 + \frac{3\mu}{(\rho_L \sigma d_L)^{1/2}}\right]^{1/6} \tag{8}$$

where μ (cp) is the viscosity, σ (dyn$\cdot$cm^{-1}) the surface tension, ρ_L (g$\cdot$cm^{-3}) the liquid density, and d_L (cm) is the ligament diameter given by Ref. 3

$$d_L = 0.9614 \left[\frac{K^2\sigma^2}{\rho\rho_L U^4}\right]^{1/6} \left[1 + 2.60\,\mu \sqrt[3]{\frac{K\rho^4 U^7}{72\,\rho_L^2 \sigma^5}}\right]^{1/5} \tag{9}$$

where ρ (g$\cdot$cm^{-3}) is the density of the gaseous medium, U can be taken nearly equal to the jet velocity,[3,4] and K is written as[4]

$$K = \frac{R^2 \sin^3}{(1 - \cos\phi \cos\theta)^2} \tag{10}$$

Then the distribution function, taken as $f(\phi) = \mathrm{d}\dot{n}/\mathrm{d}\phi$ can be written as

$$f(\phi) = \frac{6V_0 R^2 \sin\theta}{\pi d_d^3 (1 - \cos\theta\cos\phi)} \sqrt{1 - \cos^2\theta \sin^2\phi} \tag{11}$$

The moments of the above distribution function can then be calculated using Eq. (11) along with Eqs. (8–10). In particular, it is worthwhile to evaluate the several existing "mean diameters" which are used in different fields of application.

These diameters, along with their definitions and most common applications are listed in Table 1, taken from Marshall.[5]

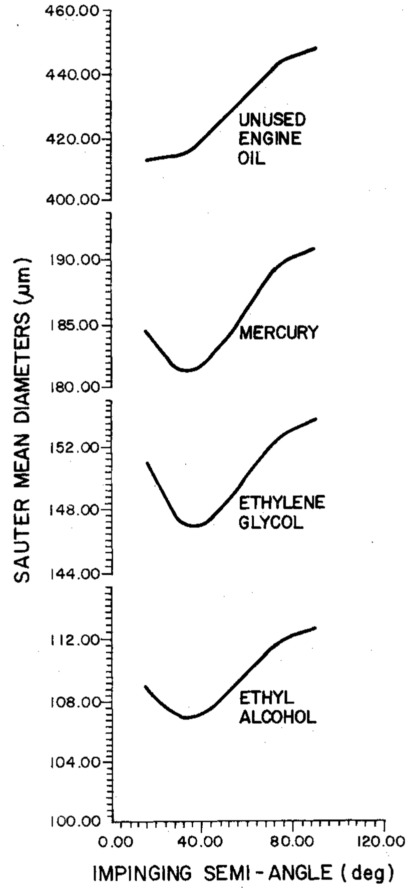

Fig. 5 Sauter mean diameter for various liquids vs impinging semiangle θ for a given configuration (R = 0.05 cm, U = 3000 cm $\cdot$ s^{-1}).

Results

Figure 2, taken from Ref. 4, shows the azimuthal droplet diameter distribution for a given configuration and various liquids, and it is built using Eqs. (8–10). These equations plus Eq. (11) allow the construction of the curves shown in Fig. 3, in the case of ethyl alcohol, for the several mean diameters as defined in Table 1, vs the impingement semiangle θ.

Equation (7), along with Eqs. (8–10), yields the droplet formation rate, $\dot{n}$, for given increments of the azimuthal angle ϕ. This is presented in Fig. 4 for a given configuration and the same conditions and liquids as those used to plot Fig. 2.

Notice that the Sauter mean diameter $\bar{X}_{vs}$ plotted in Fig. 3 for ethyl alcohol shows a minimum value for a given θ. This situation is shown more precisely in Fig. 5 along with results for other liquids.

Concluding Remarks

A very interesting feature can be seen in the Sauter mean diameter vs θ curve (Fig. 5): the ethyl alcohol, the ethylene glycol, and the mercury curves show a minimum value around θ = 35 deg, while for the unused engine oil, a point of inflection occurs for nearly the same value of θ. This suggests a useful datum to be considered in the design of the injectors considered here, i.e., the best total impinging angle for sprays generated by impinging jets is around 2θ = 70 deg. This is so because by attaining a minimum $\bar{X}_{vs}$ one guarantees a maximum fuel evaporation rate.

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